Calculus

Instructions Please write your solutions on your own paper. Explain your reasoning in complete sentences to maximize credit.

1. Suppose f is a function such that $f(0) = 1$ and $f'(0) = 3$. Let g be the composite function such that $g(x) = f(\sin x)$. Determine the value of the derivative $g'(0)$.

2. Suppose f is a function such that $f(2) = 4$ and $f'(2) = 7$. Let g be the composite function such that $g(x) = e^{f(2x)}$. Determine the value of the derivative $g'(1)$.

3. Suppose f and g are inverse functions [recall this means that $f(g(x)) = x$ and $g(f(x)) = x$], and suppose $f(1) = 2$, $f'(1) = 3$, $f(2) = 5$, and $f'(2) = 7$. Determine the value of the derivative $g'(2)$.

4. According to the TI-89 calculator, the functions $\ln(x)$ and $\ln(171x)$ have the same derivative: namely, $1/x$. Does it make sense that the graphs of these two different functions have the same slope? Explain.

5. Suppose $f(2) = 3$ and $f'(2) = 5$. Let g be the function such that $g(x) = x^{f(x)}$. Use logarithmic differentiation to determine the value of the derivative $g'(2)$.

6. Show that the curves $y = x^3$ and $x^2 + 3y^2 = 1$ intersect orthogonally.

7. Two students leave Milner Hall at the same time, walking in perpendicular directions. One student walks northeast along Ross Street at a speed of 3 feet per second, and the other student walks northwest along Asbury Street at a speed of 4 feet per second. At what rate is the distance between the students increasing 10 seconds after they start walking?

8. Consider the curve given in parametric form by $x(t) = t^3$ and $y(t) = \sin(t)$, where t runs over the real numbers. Do these parametric equations determine y as a one-to-one function of x? Explain why or why not.

9. If you invest \$1,000 at 5% interest compounded continuously, then the amount $A(t)$ in your account after t years is $A(t) = 1{,}000\, e^{t/20}$. How many years does it take to double your money?

Calculus

10. **Optional problem for extra credit**
Jackie finds an approximate value for $\sqrt[3]{1001}$ (the cube root of 1001) by using the linear approximation for the function $\sqrt[3]{x}$ with the base point $a = 1000$. Jamie finds an approximate value for $\sqrt[3]{1001}$ by doing one iteration of Newton's method applied to the function $x^3 - 1001$ with starting point $x_0 = 10$. Whose approximation to $\sqrt[3]{1001}$ is more accurate?