Calculus

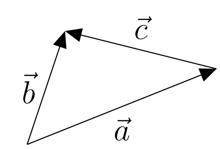
Instructions Please write your name in the upper right-hand corner of the page.

Write your solutions in complete sentences. For instance, your answer to the question "What is your favorite color?" should be not "maroon" but "My favorite color is maroon."

If you need more space to write your solutions, you may use the back of the page or a separate sheet of paper.

1. Where does our class meet on Friday?

   Solution. Our Friday meetings are in Blocker 130, which is one of the Department of Mathematics computing laboratories.



2. In the diagram, vector $\vec{a} = \langle 5, 2\rangle$ and vector $\vec{b} = \langle 1, 3\rangle$. Find the components of vector $\vec{c}$.

   Solution. According to the triangle law for addition of vectors, $\vec{b} = \vec{a} + \vec{c}$. Therefore $\vec{c} = \vec{b} - \vec{a} = \langle 1 - 5, 3 - 2\rangle = \langle -4, 1\rangle$.

3. Find a vector that is orthogonal to the vector $\langle 2, 4\rangle$. Is there more than one correct answer? Explain.

   Solution. One correct answer is the orthogonal complement of the vector $\langle 2, 4\rangle$: namely, the vector $\langle -4, 2\rangle$. The opposite vector $\langle 4, -2\rangle$ is another correct answer; in fact, any scalar multiple of $\langle -4, 2\rangle$ is a correct answer.

4. Suppose a constant force of 5 N moves an object a distance of 10 m. If the work done is equal to 25 J, what is the angle between the force vector and the displacement vector?

   Solution. The dot product of the vector force $\vec{F}$ and the vector displacement $\vec{d}$ equals the work. Thus $25 = \vec{F} \cdot \vec{d} = 5 \times 10 \times \cos\theta$, where θ is the angle between the two vectors. Therefore $\cos\theta = 1/2$. This value corresponds to a standard triangle, and you should know that θ equals 60° or $\pi/3$ radians. [You could also find θ on your calculator as $\cos^{-1}(1/2)$.]

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5. Suppose the scalar projection of vector $\vec{v}$ onto vector $\vec{w}$ is equal to the scalar projection of vector $\vec{w}$ onto vector $\vec{v}$. What deduction can you make about the vectors $\vec{v}$ and $\vec{w}$?

   Solution. The indicated scalar projections are $\vec{v} \cdot \vec{w}/|\vec{w}|$ and $\vec{w} \cdot \vec{v}/|\vec{v}|$.

   Since $\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$, the two scalar projections will certainly be equal if $|\vec{w}| = |\vec{v}|$, that is, if the two vectors have the same length.

   The other way that the two scalar projections can be equal is if $\vec{v} \cdot \vec{w} = 0$, that is, if the vectors $\vec{v}$ and $\vec{w}$ are orthogonal to each other.

   In summary, the valid deduction is that *either* the vectors $\vec{v}$ and $\vec{w}$ have the same length *or* the vectors $\vec{v}$ and $\vec{w}$ are orthogonal (or both).