Recap from yesterday

The dot product has two equivalent representations:

$$\vec{v} \cdot \vec{w} = |\vec{v}|\,|\vec{w}|\cos(\theta)$$
$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2$$

where θ is the angle between the vectors, and $\vec{v} = \langle v_1, v_2 \rangle$, and $\vec{w} = \langle w_1, w_2 \rangle$.

The component of $\vec{v}$ in the direction of $\vec{w}$ is $|\vec{v}|\cos(\theta)$, or $\vec{v} \cdot \dfrac{\vec{w}}{|\vec{w}|}$.

Follow up on projection

The quantity $\vec{v} \cdot \frac{\vec{w}}{|\vec{w}|}$ is the *scalar projection* of the vector $\vec{v}$ onto the vector $\vec{w}$. This quantity represents the component of $\vec{w}$ in the direction of $\vec{w}$.

The *vector projection* of $\vec{v}$ onto $\vec{w}$ is the vector obtained by multiplying the scalar projection by a unit vector in the direction of $\vec{w}$, that is,

$$\left(\vec{v} \cdot \frac{\vec{w}}{|\vec{w}|}\right) \frac{\vec{w}}{|\vec{w}|}$$

Example

If $\vec{v} = \langle 1, 2 \rangle$ and $\vec{w} = \langle 3, 4 \rangle$, then the scalar projection of $\vec{v}$ onto $\vec{w}$ equals $11/5$
and the vector projection equals $\langle 33/25, 44/25 \rangle$.

Parametric equations

Example

The equations

$$x = \cos(\theta), \qquad y = \sin(\theta), \qquad 0 \leq \theta \leq 2\pi$$

are *parametric equations* for a circle of radius 1 in the xy plane [since $(\cos\theta)^2 + (\sin\theta)^2 = 1$].

Using vectors, you could write $\vec{r}(\theta) = \langle \cos(\theta), \sin(\theta) \rangle$ or $\vec{r}(\theta) = \vec{\imath}\cos(\theta) + \vec{\jmath}\sin(\theta)$, where $\vec{r}$ is the *position vector* that joins the origin to a point on the curve.

Equations of lines

slope-intercept form $y = mx + b$

point-slope form $y - y_0 = m(x - x_0)$

vector form $\vec{r}(t) = \vec{r_0} + t\vec{v}$, where $\vec{r_0}$ is the position vector of some point on the line, and t is the parameter, and $\vec{v}$ is a vector parallel to the line.

Example

Find a vector equation for the line passing through the points $(1, 2)$ and $(3, 4)$.
Solution: Take $\vec{r_0}$ to be $\langle 1, 2\rangle$ and take $\vec{v}$ to be $\langle 3 - 1, 4 - 2\rangle$.
Then $\vec{r}(t) = \langle 1, 2\rangle + t\langle 2, 2\rangle$.

Assignment

- Do the odd-numbered problems 1–13 in Appendix J.3 and check your answers in Appendix L (not to hand in).

  (Alert! Some of these are hard!)

Quiz

1. Suppose $\vec{v}$ is a vector. For each of the following quantities, say if it is a vector, a scalar, or a meaningless expression.
   (a) $5\vec{v}$ (b) $5 + \vec{v}$ (c) $5 + |\vec{v}|$ (d) $|5\vec{v} \cdot \vec{v}|$

2. Suppose $\vec{v} + \vec{w} = \langle 3, -3 \rangle$ and $\vec{v} - \vec{w} = \langle -1, 3 \rangle$.
   Find $\vec{v} + 2\vec{w}$.

3. Suppose $\vec{v} = 3\vec{\imath} + 4\vec{\jmath}$, and $\vec{v} \cdot \vec{w} = 0$, and $|\vec{w}| = 1$. There are two solutions for $\vec{w}$. Find both of them.