Comment on Problem 13

Solve $x = \dfrac{1-t}{1+t}$ by high-school algebra to get $t = \dfrac{1-x}{1+x}$, and $y = t^2$, so

$$y = \left(\frac{1-x}{1+x}\right)^2 .$$

Help

- I have office hours in Blocker 601L, Monday and Wednesday afternoons, 2:00–3:00.
- Our teaching assistant, Angelique, has office hours in Blocker 221B, after class on Tuesday and Thursday 1:00–2:00 and before class on Wednesday 3:00–4:00.
- The Department of Mathematics has evening drop-in help sessions for many courses. The help session for Math 151/171 meets in Blocker 117 on Monday, Tuesday, Wednesday, and Thursday evenings, 5:00–7:30.

Limits: examples

Summary of the class discussion:

We looked in desmos at graphs of $\cos(x) + x\sin(1/x)$ and $\cos(1/x)$ and $x\ln(x)\sin(1/x)$ and $|x| + \cos(\pi/x)$ to see what can be said from a graph about limits when $x \to 0$.

The first example has limit 1; the second example has no limit; the third example has a limit from the right (symbolized by $\lim_{x\to 0^+}$); and the fourth example seemed, from a table of data, to have a limit, but the graph shows that actually there is no limit.

Limits: the easy case

When is $\lim_{x \to b} f(x) = f(b)$?

- If $f(x)$ is a polynomial, like $7x^5 - 3x^3 + \frac{2}{9}x - \sqrt{\pi}$.
- If $f(x)$ is a rational function (a quotient of polynomials), like $\dfrac{5x^3 - 2x + 1}{x^2 + 7}$, as long as the denominator is not equal to zero at b.
- If $f(x)$ is an exponential function, a logarithm function, or a trigonometric function, as long as b is in the domain of the function.

Limits: examples with holes in the domain

- $\lim_{x\to 2} \frac{x^2-4}{x^2-x-2}$

  Factor: $\frac{x^2-4}{x^2-x-2} = \frac{(x-2)(x+2)}{(x-2)(x+1)} = \frac{x+2}{x+1}$, so the limit as $x \to 2$ equals $4/3$.

  Notice that $\frac{x+2}{x+1} = 1 + \frac{1}{x+1}$, so the graph looks like the graph of $1/x$ but shifted 1 unit to the left and 1 unit up. The graph of the original function has a hole at the point where $x = 2$ and $y = 4/3$.

Assignment (not to hand in)

- Do the odd-numbered problems 5–11 in section 2.2 and check your answers in the back of the book.
- Do problems 21, 23, and 25 in Appendix J.2 and check your answers in Appendix L.