Math Club Meeting February 5 (today) 7:00pm Blocker 220 Prof. Jay Walton will speak about mathematical ecology. Also on the agenda: pizza.

Tangent lines

Two points are needed to determine a line, but a tangent line might touch a graph at only one point.

Strategy: Find the slope of a *secant* line through two points near each other on the graph; take a limit as the points come together.

Example

When $f(x) = x^2 + 1$, find the slope of the line tangent to the graph at the point (2, f(2)) = (2, 5).

Solution: Compute
$$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{(x^2 + 1) - (2^2 + 1)}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x \to 2} (x + 2) = 4.$$

The equation of the tangent line is y-5 = 4(x-2), or y = 4x-3.

Notation and terminology

The limit $\lim_{x\to b} \frac{f(x) - f(b)}{x - b}$ or, equivalently, $\lim_{h\to 0} \frac{f(b + h) - f(b)}{h}$ is denoted by the symbol f'(b), called the *derivative* of f at b.

The meaning of f'(b) is the slope of the line tangent to the graph of f at the point (b, f(b)), equivalently, the instantaneous rate of change of f at b. Assignment (not to hand in)

▶ In Section 2.6, Exercises 3, 7, 19, 27, 35, 39, 49, 67.