About Exam 1

- Exam 1 takes place in class this week on Thursday, February 14.
- Material covered on the exam: Chapter 2 (limits and derivatives) and Appendix J (vectors and vector functions).
- All of the exam problems are work-out problems.
- Please bring your own paper to the exam.

Recap on limit definitions of the derivative

A convenient formula for the derivative at a specific value b is
$f'(b) = \lim_{x \to b} \dfrac{f(x) - f(b)}{x - b}$.

A convenient formula for the derivative at a general value x is
$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

Various notations for the derivative:
$f'(x)$ and $\dfrac{df}{dx}$ and $\dfrac{d}{dx}f(x)$ and $Df(x)$ all mean the same thing.

Example: an exponential function

If $f(x) = 2^x$, what is $f'(x)$?

$$\lim_{h\to 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h\to 0} \frac{2^x 2^h - 2^x}{h} = \lim_{h\to 0} 2^x \left(\frac{2^h - 1}{h}\right)$$
$$= 2^x \quad \text{times a constant.}$$

The slope of the graph of this function is proportional to the height of the graph!

A similar calculation applies to 3^x or 4^x or any b^x:
the derivative is a constant times the function.

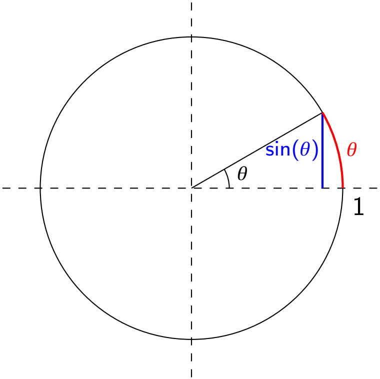
"The" exponential function

There is a base b for which the derivative of the function b^x is equal to itself.

That special base is e, named by the Swiss mathematician Leonhard Euler (1707–1783).

$$\boxed{\frac{d}{dx}e^x = e^x}$$

Derivative of the sine function at 0



When θ is measured in *radians*, the length of the red arc equals θ.

The picture makes it plausible that $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$.

Interpretation as a derivative: $\sin'(0) = 1$.

A key formula: $\boxed{\frac{d}{dx}\sin(x) = \cos(x)}$

The computation:

$$\lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x)\cos(h)+\cos(x)\sin(h)-\sin(x)}{h}$$

(by a trigonometric identity)

$$= \lim_{h\to 0} \sin(x)\frac{\cos(h)-1}{h} + \lim_{h\to 0} \cos(x)\frac{\sin(h)}{h}$$

$$= \sin(x)\cos'(0)+\cos(x)\sin'(0)$$

$$= \cos(x).$$

Assignment: Study for the exam

Some suggestions:

- Work on the Review at the end of Appendix J and at the end of Chapter 2.
- Work on the old exam.
- Study in groups.
- Go to help session.