Related rates

Example

If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm. [Exercise 14 in Section 3.9]

- Introduce variables: S = surface area; D = diameter.
- Write a relation between the variables: $S = 4\pi r^2 = \pi D^2$.
- Take the derivative with respect to time, using the chain rule: $\dfrac{dS}{dt} = 2\pi D \dfrac{dD}{dt}$.
- Plug in the numbers: $-1 = 2\pi \times 10 \times \dfrac{dD}{dt}$.
- Solve for the unknown derivative: $\dfrac{dD}{dt} = \dfrac{-1}{20\pi}$ cm/min.

Linear approximation (tangent line approximation)

If $y = f(x)$, then the tangent line to the graph at the point where $x = b$ and $y = f(b)$ is
$y - f(b) = f'(b)(x - b)$ or $y = f(b) + f'(b)(x - b)$.
If x is close to b, then $\boxed{f(x) \approx f(b) + f'(b)(x - b)}$

Example

Find an approximate value for $\frac{1}{10.2}$ (evidently a bit less than 0.1).

Solution

If $f(x) = \frac{1}{x}$, then $f'(x) = -\frac{1}{x^2}$. Take b to be 10, then $f'(b) = -\frac{1}{100}$.
Now take x to be 10.2. The linear approximation formula says
$f(10.2) \approx f(10) + f'(10)(0.2) = 0.1 - \frac{1}{100}(0.2) = 0.1 - 0.002 = 0.098$

Assignment (not to hand in)

- In Section 3.9, Exercises 3, 5, 7, 9, 13, 17, 21.
- In Section 3.10, Exercises 1, 3, 11, 23, 27, 31, 33.