Reminders

- Our last class meeting is Thursday, April 25 (because Tuesday, April 30 is redefined as Friday).
- The comprehensive final exam takes place 3:00–5:00 in the afternoon of Thursday, May 2.

Antiderivatives: the guess-and-check method

Example

Find an antiderivative of $\sqrt{2x+3}$.

The power rule suggests that the answer might be $k(2x+3)^{3/2}$ for some constant k. So differentiate this candidate antiderivative to see if you can choose a workable value for k:

$$\frac{d}{dx}k(2x+3)^{3/2} = k\cdot\frac{3}{2}(2x+3)^{1/2}\cdot\frac{d}{dx}(2x+3) = 3k(2x+3)^{1/2},$$

so it checks if $k = 1/3$.

So the answer to the original problem is $\frac{1}{3}(2x+3)^{3/2} + C$.

Antiderivatives: the substitution method

Example

Compute the antiderivative $\int x\sqrt{1+x^2}\,dx$.

Solution: Introduce a new variable u as follows.

$$u = 1 + x^2, \qquad \frac{du}{dx} = 2x,$$
$$du = 2x\,dx, \qquad \frac{1}{2}\,du = x\,dx.$$

The problem becomes $\int \sqrt{u}\,\frac{1}{2}\,du$ or $\frac{1}{2}\int u^{1/2}\,du$. By the power rule, the answer is $\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$ or $\frac{1}{3}(1+x^2)^{3/2} + C$.

Exercises for this week (not to hand in)

- Section 5.4: Exercises 7, 9, 11, 21, 23, 25, 27, 29, 33, 45, 49.
- Section 5.5: Exercises 1, 3, 5, 7, 11, 13, 17, 21, 25, 27, 33, 53, 59, 67, 69.