Quiz solutions

January 24, 2019

1. Determine the vector projection of the vector $3\vec{\imath} + \vec{\jmath}$ onto the vector $\vec{\imath} + 2\vec{\jmath}$.

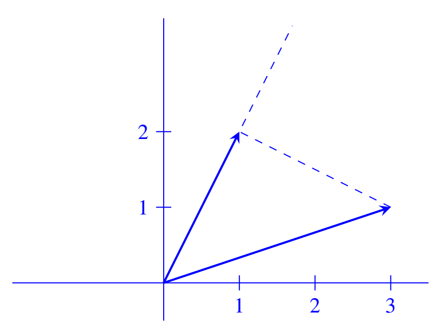
   Solution. Since $|\vec{\imath} + 2\vec{\jmath}| = \sqrt{1^2 + 2^2} = \sqrt{5}$, a unit vector in the direction of $\vec{\imath} + 2\vec{\jmath}$ is $\dfrac{\vec{\imath}}{\sqrt{5}} + \dfrac{2\vec{\jmath}}{\sqrt{5}}$. The *scalar* projection of the vector $3\vec{\imath} + \vec{\jmath}$ onto the vector $\vec{\imath} + 2\vec{\jmath}$ is therefore equal to the dot product

   $$(3\vec{\imath} + \vec{\jmath}) \cdot \left(\frac{\vec{\imath}}{\sqrt{5}} + \frac{2\vec{\jmath}}{\sqrt{5}}\right), \quad \text{or} \quad \frac{3}{\sqrt{5}} + \frac{2}{\sqrt{5}}, \quad \text{or} \quad \frac{5}{\sqrt{5}}, \quad \text{or} \quad \sqrt{5}.$$

   The *vector* projection is this scalar multiplied times the already determined unit vector:

   $$\sqrt{5}\left(\frac{\vec{\imath}}{\sqrt{5}} + \frac{2\vec{\jmath}}{\sqrt{5}}\right), \quad \text{or} \quad \boxed{\vec{\imath} + 2\vec{\jmath}}.$$

   Remark. A surprise is that the answer turns out to be equal to one of the original vectors. The explanation is seen in the diagram: a perpendicular dropped from the head of the vector $\langle 3, 1\rangle$ to the line with slope 2 hits that line precisely at the head of the vector $\langle 1, 2\rangle$.



2. Compute the value of $\lim_{x \to -3} \frac{x^2+3x}{x^2-x-12}$.

Solution. Factor and cancel. If $x \neq -3$, then

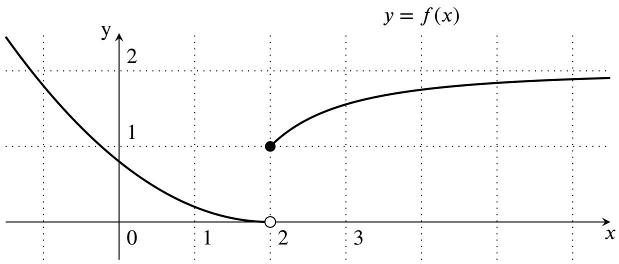
$$\frac{x^2+3x}{x^2-x-12} = \frac{x(x+3)}{(x-4)(x+3)} = \frac{x}{x-4}.$$

Therefore

$$\lim_{x \to -3} \frac{x^2+3x}{x^2-x-12} = \lim_{x \to -3} \frac{x}{x-4} = \frac{-3}{-3-4} = \boxed{\frac{3}{7}}.$$

Remark. This problem is Exercise 12 in Section 2.3 of the textbook.

3. In the graph below, which of the values $\lim_{x \to 0} f(x)$ and $\lim_{x \to 2^+} f(x)$ and $\lim_{x \to \infty} f(x)$ is the largest?



Solution. Inspecting the graph reveals that $\lim_{x \to 0} f(x)$ is some positive number less than 1, and $\lim_{x \to 2^+} f(x) = 1$. Since only a portion of the graph is shown, there is no way to be absolutely certain about the value of $\lim_{x \to \infty} f(x)$, but the picture strongly suggests that $\lim_{x \to \infty} f(x) = 2$. Under this assumption, the largest of the three limits is $\boxed{\lim_{x \to \infty} f(x)}$.