True or false?

This sentence is false.
Apparently, the sentence is both true and false; or neither.
So we have to be careful about language.

Introductions

- Who are you?
- What do you do for fun?
- When will you graduate?
- Where are you from?
- Why are you studying mathematics?

Vocabulary for today

- Statement
- Open sentence
- Universal quantifier
- Existential quantifier
- Negation

Statements

A statement is a sentence that is either true or false.
Examples:

- The number π is irrational.
- The instructor's shirt is red.

Non-examples:

- Are we having fun yet?
  (questions are not statements)
- Go for the gold.
- $x^2 = 2x$
  (an *open sentence* that becomes a statement only when a specific value is substituted for the variable)

Quantifiers

Example: "there exists an x for which $x^2 = 2x$" is a quantified statement with an existential quantifier.
In symbols, $\exists x\ x^2 = 2x$.
(True statement: $x = 0$ works, and so does $x = 2$.)
Example: "For every x, $x^2 = 2x$" is a quantified statement with a universal quantifier.
In symbols, $\forall x\ x^2 = 2x$.
(False statement: the equation is wrong when $x = 1$, for example.)
Examples with implicit quantifiers:

- Aggies love traditions.
  (implicit universal quantifier)
- Some weeks have eight days.
  (implicit existential quantifier)

Negation

If $P(x)$ is the open sentence $x^2 = 2x$, then the negation is $x^2 \neq 2x$, symbolized by $\neg P(x)$.
Interaction of negation with quantifiers:
$\neg(\exists x\ x^2 = 2x)$ means the same as $\forall x\ x^2 \neq 2x$.
Similarly, $\neg(\forall x\ x^2 = 2x)$ means the same as $\exists x\ x^2 \neq 2x$.

Examples from page 12

- The area of a rectangle is its length times its width.
  (Universal: For every rectangle, the area is the length times the width.)
- A triangle may be equilateral.
  (Existential: There exists an equilateral triangle.)