Building complicated statements from simpler ones

- Example: "Today is Tuesday, and it's sunny."
  P and Q, symbolized by $P \wedge Q$, is *conjunction*.
- Example: "Today is Tuesday, or it's sunny."
  P or Q, symbolized by $P \vee Q$, is *disjunction*.

In mathematics, "or" is inclusive: one or the other or possibly both.

- The open sentence $|x| < 5$ is a conjunction: namely, $x < 5$ and $x > -5$.
  Negation is the disjunction $x \geq 5$ or $x \leq -5$.
- The open sentence $|x| > 5$ is a disjunction: namely, either $x > 5$ or $x < -5$.

Negating compound statements

- $\neg(P \wedge Q)$ means "it is not the case that both P and Q are true," so either P or Q is false (or both are false), that is, $(\neg P) \vee (\neg Q)$.
- $\neg(P \vee Q)$ means "not either P or Q is true," that is, neither P nor Q is true, so $(\neg P) \wedge (\neg Q)$.

Truth tables

A way to understand a complicated statement, like

$$(P \wedge \neg Q) \vee (Q \wedge \neg P),$$

is to make a table of truth values.

P	Q	$\neg Q$	$\neg P$	$(P \wedge \neg Q)$	$(Q \wedge \neg P)$	$(P \wedge \neg Q) \vee (Q \wedge \neg P)$
T	T	F				
T	F	T				
F	T	F				
F	F	T				

The completed table

P	Q	$\neg Q$	$\neg P$	$(P \wedge \neg Q)$	$(Q \wedge \neg P)$	$(P \wedge \neg Q) \vee (Q \wedge \neg P)$
T	T	F	F	F	F	F
T	F	T	F	T	F	T
F	T	F	T	F	T	T
F	F	T	T	F	F	F

This table represents "exclusive or": $(P \wedge \neg Q) \vee (Q \wedge \neg P)$ is true when exactly one of P and Q is true.

Assignment

Exercises 8 and 14 on pages 26–27 to hand in next time.