

## Reminder

Exam 1 takes place in class on Tuesday, February 21.  
The exam covers Chapters 1 and 2.

# Building new sets from old

- ▶ Cartesian product
- ▶ Power set
- ▶ Partitions

# Cartesian product $A \times B$



René Descartes  
(1596–1650)

$A \times B$  means the set of ordered pairs  $(a, b)$ , where  $a \in A$  and  $b \in B$ .

Example:  $A = \{1, 2\}$ ,  $B = \{\alpha, \beta\}$ ; then  
 $A \times B = \{(1, \alpha), (1, \beta), (2, \alpha), (2, \beta)\}$ .

Note:  $A \times B \neq B \times A$ .

Cardinality works:  $|A \times B| = |A| \times |B|$ .

## Power set $\mathbf{P}(A)$

When  $A$  is a set, the *power set*  $\mathbf{P}(A)$  is the set of all subsets of  $A$ .

Example:  $A = \{4, 5, 7\}$ .

Power set

$\mathbf{P}(A) = \{\{4\}, \{7\}, \{5\}, \{4, 7\}, \{4, 5\}, \{5, 7\}, \{4, 5, 7\}, \emptyset\}$ .

In general,  $|\mathbf{P}(A)| = 2^{|A|}$ .

# Partitions

A *partition* of a set  $A$  is a collection of non-empty subsets of  $A$  that are pairwise disjoint and whose union equals  $A$ .

Example:  $A = \{4, 5, 7\}$ .

One partition is the pair of subsets  $\{5, 7\}$  and  $\{4\}$ .

Another partition is  $\{5\}$  and  $\{4, 7\}$ .

Another partition is  $\{7\}$  and  $\{4, 5\}$ .

Another partition is three singletons:  $\{4\}$ ,  $\{5\}$ , and  $\{7\}$ .

Another partition is the whole set itself.

## Pigeonhole principle (Dirichlet's drawer principle)



Peter Gustav Lejeune Dirichlet  
(1805–1859)

Example: In a set of 8 people, some 2 of them must have been born on the same day of the week.

Example: In a set of 15 people, at least 3 must have been born on the same day of the week.