Exam results

- Grading: 14 points for each of 7 problems, plus 2 points for free
- Mean 84, median 85
- Two scores of 100
- Congratulations on all your hard work!

Vocabulary today

- surjective
- injective
- bijective
- permutation

Surjective

If $f : A \to B$ is a function, then f is *surjective* (adjective) or *a surjection* (noun) if the image fills up the whole codomain.

Example. $A = \mathbf{R}$, $B = \mathbf{R}$, $f(x) = x$ (the identity function).

$A = \mathbf{R} = B$. $f(x) = x^2$. Not surjective, because all negative numbers are missing from the image.

$A = \mathbf{R}$, $B =$ the real numbers that are bigger than or equal to zero, $f(x) = x^2$. Now the image equals the codomain, so the new function is surjective.

$A = \mathbf{Z}$, $B =$ positive integers Z^+, $f(n) = |n|$. Not a well defined function, because $f(0)$ is not an element of the codomain.
One possible fix is to change the domain A to all nonzero integers, $\mathbf{Z} - \{0\}$. Then the function is well defined and surjective.

Injective

If $f : A \to B$ is a function, then f is *injective* (adjective) or *an injection* (noun) if distinct elements of the domain have distinct images.
That is, if $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$.
The identity function is both injective and surjective.

Example. $A = \mathbf{Z} - \{0\}$, $B = \mathbf{Z}^+$, $f(n) = |n|$. This function is surjective but not injective.

Bijective

If $f : A \to B$ is a function, then f is *bijective* (adjective) or *a bijection* (noun) if f is simultaneously injective and surjective.

If $A = B$, then a bijection can be called a *permutation*.