Examination 1

Instructions: Please write your solutions on your own paper. These problems should be treated as essay questions to answer in complete sentences.

1. Consider the following statement:

   Every polynomial of degree 3 has at least one real root.

   a) Identify the quantifiers (universal and existential) in the statement.

   b) Write the negation of the statement.

2. Let $P(x)$ be the open sentence "$x \in A$," and let $Q(x)$ be the open sentence "$x \in B$." Suppose $A \subseteq B$ (that is, A is a subset of B). Complete the sentence

   $Q(x)$ is a ____________ condition for $P(x)$

   by filling in the blank with the appropriate word ("necessary" or "sufficient").

3. Rewrite the implication

   $$P \Rightarrow Q$$

   in a logically equivalent form using only the symbols $\neg$ and $\wedge$ (negation and conjunction), the letters P and Q, and parentheses. Explain your reasoning.

4. Consider the following true statement:

   If n is an even integer and m is an odd integer, then the product nm is an even integer.

   a) Write the contrapositive of the statement. Is it true? Explain why or why not.

   b) Write the converse of the statement. Is it true? Explain why or why not.

5. Suppose $A = \{ n \in \mathbb{Z} \mid n^3 \geq 5 \}$, and $B = \{ n \in \mathbb{Z} \mid n \geq 0 \}$. Which of the two sets $B - A$ (the complement of A in B) and $A - B$ (the complement of B in A) has more elements? Explain your reasoning.

6. State De Morgan's laws about complements of unions and intersections.

7. Call a partition of a set *uniform* if all the subsets that are elements of the partition have the same cardinality as each other. For example, if $A = \{1, 2, 3\}$, then the set A admits precisely two uniform partitions: namely, the partition into three singleton sets, and the trivial partition whose only element is the set A itself.

   How many uniform partitions does the set $\{1, 2, 3, 4\}$ have? Explain your reasoning.