Final Examination

Instructions. Please write your solutions on your own paper. These problems should be treated as essay questions to answer in complete sentences.

1. Consider the statement: "Every Aggie wears maroon," or (equivalently) "If x is an Aggie, then x wears maroon." Write
   a) the converse of the statement;
   b) the contrapositive of the statement.

2. Does the operation of taking the Cartesian product of sets distribute over the operation of taking the intersection of sets? In symbols, can you say that

$$A \times (B \cap C) = (A \times B) \cap (A \times C) \qquad \text{for all sets } A, B, \text{ and } C\,?$$

   Explain why or why not.

3. Suppose a function $f : \mathbb{Z} \to \mathbb{Z}$ is defined as follows: $f(n) = 3n + 1$ for each integer n.
   a) Is the function f injective?
   b) Is the function f surjective?

   Explain why or why not.

4. Suppose a relation is defined on subsets of the integers by saying that set A is related to set B when A is a subset of B (in symbols, $A \subseteq B$).
   a) Is this relation reflexive?
   b) Is this relation symmetric?
   c) Is this relation transitive?

   Explain why or why not.

5. Here are five concepts from the course that start with the letter p:
   a) partial ordering on a set
   b) partition of a set
   c) permutation of a set
   d) pigeonhole principle
   e) power set of a set

   Explain the meaning of **three** of these concepts.

6. Use the method of mathematical induction to prove *Bernoulli's inequality*: namely, if x is a real number greater than -1, and n is a positive integer, then $(1 + x)^n \geq 1 + nx$. (Treat the number x as a fixed quantity and the number n as the induction variable.)

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Bonus problem. The definition of "f is continuous at c" that you will learn in Math 409 says: "For every positive ε, there exists a positive δ such that $|f(x) - f(c)| < \varepsilon$ whenever $|x - c| < \delta$." Write the negation of this statement without using the word "not."

Hint: There is an implicit universal quantifier hidden in the word "whenever."