Math 221 Examination 2 Spring 2013

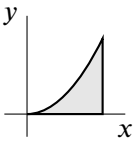
# Several Variable Calculus

**Instructions** These problems should be viewed as essay questions. Before making a calculation, you should explain in words what your strategy is.

Please write your solutions on your own paper. Each of the 10 problems counts for 10 points.

1. Evaluate the iterated integral $\displaystyle\int_0^{\pi/2}\int_0^1 x\cos(xy)\,\mathrm{d}y\,\mathrm{d}x$.

2. Describe the solid whose volume is given by the spherical-coordinate triple integral
$$\int_0^{\pi}\int_0^{\pi}\int_1^2 \rho^2\sin(\phi)\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta.$$
(You do not need to evaluate the integral.)

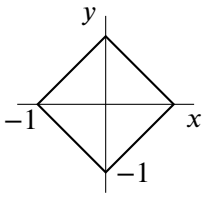



3. Evaluate the double integral $\displaystyle\iint_D xe^y\,\mathrm{d}A$, where $D$ is the region in the first quadrant bounded by the line $y = 0$, the line $x = 1$, and the parabola $y = x^2$.

4. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $x + z = 2$.

5. Rewrite the integral $\displaystyle\int_0^2\int_0^y\int_0^{y^2} f(x,y,z)\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$ as an iterated integral in the order $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.

6. Set up an integral for the surface area of the part of the paraboloid $z = x^2+y^2$ that lies between the plane $z = 1$ and the plane $z = 4$.
(You do not need to evaluate the integral.)




7. Let $D$ be the square with vertices $(1,0)$, $(0,1)$, $(-1,0)$, and $(0,-1)$. Rewrite the double integral $\iint_D (x+y)^5\,\mathrm{d}A$ as an integral with respect to $\mathrm{d}u\,\mathrm{d}v$, where $u = x + y$ and $v = x - y$. (You do not need to evaluate the integral.)

8. Find the work done by the force field $\vec{F}(x,y) = y^2\,\hat{\imath} + 2xy\,\hat{\jmath}$ on a particle that moves in a straight line from the point $(1,2)$ to the point $(5,1)$.

9. Evaluate the line integral $\displaystyle\int_C y\,\mathrm{d}s$ when the parametric equations of $C$ are $x = t^2$ and $y = 2t$, where $0 \le t \le 1$.

10. What does it mean to say that a vector field $\vec{F}$ is a *conservative* vector field?

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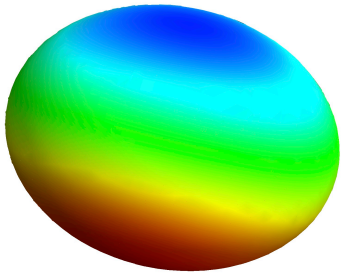
## Optional bonus problem for extra credit



Find the volume of an egg whose eggshell has the equation

$$\frac{x^2}{8} + \frac{y^2}{5} + \frac{z^2}{3} = 1.$$