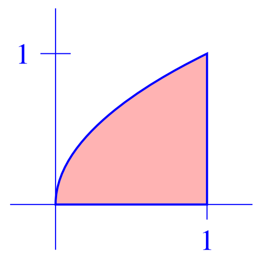
Several Variable Calculus

1. Evaluate the iterated integral $\int_0^1 \int_{y^2}^1 4y\, e^{x^2}\, \mathrm{d}x\, \mathrm{d}y$.
   Hint: Reverse the order of integration.

   Solution. The indicated bounds on x are the parabola $x = y^2$ (opening sideways) and the vertical line $x = 1$. Here is a picture of the integration region:

   Reversing the limits leads to the following integral:

$$\begin{aligned}\int_0^1 \int_0^{\sqrt{x}} 4y\, e^{x^2}\, \mathrm{d}y\, \mathrm{d}x &= \int_0^1 \left[2y^2\right]_0^{\sqrt{x}} e^{x^2}\, \mathrm{d}x \\ &= \int_0^1 2x\, e^{x^2}\, \mathrm{d}x \\ &= \left[e^{x^2}\right]_0^1 \\ &= e - 1.\end{aligned}$$

2. Evaluate the double integral $\iint_D x^2\, \mathrm{d}A$, where D is the region in the first quadrant where $x^2 + y^2 \leq 1$.

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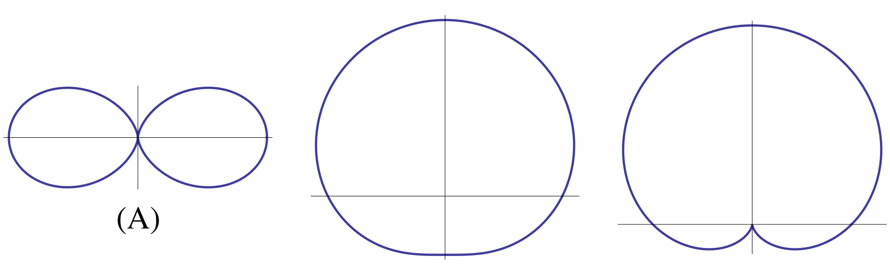
Solution. Converting to polar coordinates is the simplest approach:

$$\begin{aligned}
\iint_D x^2\,\mathrm{d}A &= \int_0^{\pi/2}\int_0^1 (r\cos\theta)^2\, r\,\mathrm{d}r\,\mathrm{d}\theta \\
&= \int_0^{\pi/2}\int_0^1 r^3\cos^2\theta\,\mathrm{d}r\,\mathrm{d}\theta \\
&= \int_0^{\pi/2}\left[\frac{1}{4}r^4\right]_0^1\cos^2\theta\,\mathrm{d}\theta \\
&= \frac{1}{4}\int_0^{\pi/2}\cos^2\theta\,\mathrm{d}\theta \\
&= \frac{1}{8}\int_0^{\pi/2}(1+\cos 2\theta)\,\mathrm{d}\theta \\
&= \frac{1}{8}\left[\theta+\frac{1}{2}\sin 2\theta\right]_0^{\pi/2} \\
&= \frac{1}{8}\left[\left(\frac{\pi}{2}+\frac{1}{2}\sin\pi\right)-\left(0+\frac{1}{2}\sin 0\right)\right] \\
&= \frac{\pi}{16}.
\end{aligned}$$

3. The polar equation $r = 2 + \sin\theta$ corresponds to which one of the following graphs? Explain how you know.



(A) (B) (C)

Solution. Observe that $1 \le 2 + \sin\theta \le 3$ (since $-1 \le \sin\theta \le 1$), so r is never equal to 0. Graph (B) is the only one that does not pass through the origin.