Math 304

Linear Algebra

Harold P. Boas

boas@tamu.edu

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Highlights

From last time:

- linear transformations

Today:

- matrix representations of linear transformations

Example

A certain linear operator L on R^2 has the following action:

input: output:

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Determine L.

Since $L\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$ and $L\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$, the transformation L is represented by multiplication by the matrix $\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$ whose columns are the images under L of the standard basis vectors.

Example continued

Composing the preceding operator with a reflection and a rotation produces a new transformation T that takes

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Find a matrix that represents the new transformation T.

Example modified for a nonstandard basis

The previous cases represented transformations with respect to the standard basis.

Consider a nonstandard basis $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{u_2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

- What matrix A transforms the $\mathbf{u}$-coordinates of a vector $\mathbf{x}$ into the standard coordinates of the image $T(\mathbf{x})$?
- What matrix B transforms the $\mathbf{u}$-coordinates of a vector $\mathbf{x}$ into the $\mathbf{u}$-coordinates of the image $T(\mathbf{x})$?
- What matrix C transforms the $\mathbf{u}$-coordinates of a vector $\mathbf{x}$ into the $\mathbf{v}$-coordinates of the image $T(\mathbf{x})$, where $\mathbf{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$?