Linear Algebra

Write your **name**: ________________________________ (2 points).

In **problems 1–5**, circle the correct answer. (5 points each)

1. Every linear system of three equations in four unknowns is consistent. `True` `False`
2. Three vectors in R^2 are always linearly dependent. `True` `False`
3. If A is a square matrix such that $\det(A) = 0$, then the homogeneous system $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions. `True` `False`
4. If $\mathbf{v}_1$, $\mathbf{v_2}$, and $\mathbf{v}_3$ are elements of a vector space V, then the span of the vectors $\mathbf{v}_1$, $\mathbf{v_2}$, and $\mathbf{v}_3$ is a subspace of V. `True` `False`
5. If A is a square matrix that is singular, then the matrix A^T (that is, the transpose of A) is singular too. `True` `False`

In **problems 6–9**, fill in the blanks. (7 points per problem)

6. $\begin{pmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & \square \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 15 & 19 \\ \square & 0 \end{pmatrix}$

7. $\det \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 1 & 1 & \square & 3 \end{pmatrix} = 18$

8. If $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$ is the end stage of the Gauss-Jordan reduction algorithm applied to the augmented matrix of a system of linear equations, then the system of equations has ______________________ solution(s). [none, exactly one, or infinitely many?]

9. Vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ form a *basis* for the vector space V if and only if $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly independent and ____________________ .

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In **problems 10–12**, show your work and explain your method. Continue on the back if you need more space. (15 points each)

10. If $A = \begin{pmatrix} 1 & 6 & 3 \\ 3 & 24 & 9 \\ 0 & 24 & 9 \end{pmatrix}$, find a lower triangular matrix L and an upper triangular matrix U such that $A = LU$.

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11. Can every polynomial of degree 2 or less be written as a linear combination of the three polynomials $1 + x - x^2$, $1 - x$, and $1 + x^2$? Explain why or why not.

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12. Write down the 2×3 matrix whose first row is the first three digits of your student identification number and whose second row is the last three digits of your student identification number. Then determine the nullspace of your matrix.