# Linear Algebra

Write your **name**: ______________________________ (2 points).

In **problems 1–5**, circle the correct answer. (5 points each)

1. On the vector space of polynomials, differentiation is a linear operator.
   `True` `False`

2. If the linear system $A\mathbf{x} = \mathbf{b}$ is consistent, then the vector $\mathbf{b}$ must be in the space $N(A)^{\perp}$. `True` `False`

3. The matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is the matrix representation (with respect to the standard basis) of the linear operator that reflects each vector $\mathbf{x}$ in $R^2$ about the $x_2$ axis and then rotates it 90° in the counterclockwise direction. `True` `False`

4. The two functions $\sqrt{3}\,x$ and $\sqrt{5}\,(4x^2 - 3x)$ are an orthonormal set in the space $C[0,1]$ with inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)\,dx$.
   `True` `False`

5. Every invertible matrix is diagonalizable. `True` `False`

In **problems 6–9**, fill in the blanks. (7 points per problem)

6. The matrix $\begin{pmatrix} \square & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \square & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is an orthogonal matrix.

7. The angle between the vectors $\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ \square \end{pmatrix}$ in $R^3$ is 45°.

8. The eigenvalues of the matrix $\begin{pmatrix} 2 & 4 \\ 3 & \square \end{pmatrix}$ are 0 and $\square$.

9. If a $7 \times 11$ matrix $A$ has a nullspace of dimension 5, then the nullspace of the transpose matrix $A^T$ has dimension $\square$.

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In **problems 10–12**, show your work and explain your method. Continue on the back if you need more space. (15 points each)

10. Suppose $\mathbf{u}_1 = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, and $\mathbf{v}_2 = \begin{pmatrix} 3 \\ 8 \end{pmatrix}$. If $\mathbf{x} = 4\mathbf{u}_1 + 3\mathbf{u}_2$, find numbers $c_1$ and $c_2$ such that $\mathbf{x} = c_1\mathbf{v_1} + c_2\mathbf{v}_2$.

11. Find a least-squares solution of the system $\begin{pmatrix} 1 & 2 \\ -1 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$.

12. The matrices $\begin{pmatrix} 2 & a & -9 \\ -4 & 2 & -6 \\ -2 & -5 & 3 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ are similar. Find the value of the number $a$.