Linear Algebra

1. Are the vectors $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$, and $\begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$ a spanning set for R^3? Explain why or why not. [This is exercise 10d on page 133.]

   Solution. This problem is closely related also to exercise 2d on p. 144. There are various ways to see that these three vectors are not a spanning set for R^3.

   One way is to observe that the second vector is the negative of the first vector, and the third vector is twice the first vector. Thus the span of the three vectors is the same as the span of the first vector. In other words, the span of the vectors is only a line—not the whole space R^3.

   Another way is to set up the augmented matrix

$$\left(\begin{array}{rrr|c} 2 & -2 & 4 & a \\ 1 & -1 & 2 & b \\ -2 & 2 & -4 & c \end{array}\right),$$

   row reduce, and observe that the corresponding linear system is inconsistent for certain values of a, b, and c. For example, the system is inconsistent when $a = 1$, $b = 0$, and $c = 0$. (In fact, the system is inconsistent unless (a, b, c) is a multiple of $(2, 1, -2)$.)

2. Is the set of odd functions in $C[-1, 1]$ a subspace of the vector space $C[-1, 1]$? Explain why or why not.

   (Recall that a function f is called odd if $f(-x) = -f(x)$ for all x, and the notation $C[-1, 1]$ means the space of continuous functions on the closed interval $[-1, 1]$.) [This is exercise 6b on page 132.]

   Solution. Our subset of the vector space $C[-1, 1]$ is closed under addition (the sum of two odd functions is odd) and closed under multiplication by scalars (a scalar multiple of an odd function is odd). Therefore our set is a subspace of $C[-1, 1]$.