Linear Algebra

Write your **name**: ______________________ (2 points).

In **problems 1–5**, circle the correct answer. (5 points each)

1. If A is a 3×3 matrix, then A is a singular matrix if and only if the linear system $A\mathbf{x} = \mathbf{0}$ is inconsistent. `True` `False`

2. If A is an invertible 3×3 matrix, then $(A^T)^{-1} = (A^{-1})^T$. `True` `False`

3. If $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are elements of a vector space V, then the span of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ (that is, the set of all linear combinations of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$) is a subspace of V. `True` `False`

4. A linear system $A\mathbf{x} = \mathbf{b}$ is consistent if and only if the column vector $\mathbf{b}$ can be written as a linear combination of the column vectors of the matrix A. `True` `False`

5. The polynomials $1 + x$, $1 + x^2$, and $2 + x + x^2$ form a basis for the three-dimensional vector space P_3 (the vector space of polynomials of degree less than 3). `True` `False`

In **problems 6–9**, fill in the blanks. (7 points per problem)

6. If $A = \begin{pmatrix} \square & 0 \\ 0 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 5 & \square \end{pmatrix}$, then $2A+B = \begin{pmatrix} 7 & \square \\ 5 & 2 \end{pmatrix}$.

7. $$\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & \square \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ \square & 1 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}$$

8. If A is an $n \times m$ matrix, then the set of all solutions to the homogeneous system $A\mathbf{x} = \mathbf{0}$ is called the ______________________ of A.

9. If $\left(\begin{array}{cccc|c} 1 & 2 & 0 & 5 & 4 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right)$ is the end stage of the Gauss-Jordan reduction algorithm applied to the augmented matrix of a system of linear equations, then the system of equations has ______________________ solution(s). [none, exactly one, or infinitely many?]

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In **problems 10–12**, show your work and explain your method. Continue on the back if you need more space. (15 points each)

10. If $A = \begin{pmatrix} 1 & 3 & 0 \\ 6 & 19 & 4 \\ 0 & 8 & 33 \end{pmatrix}$, find a lower triangular matrix L and an upper triangular matrix U such that $A = LU$.

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11. Determine the value of a for which

$$\det \begin{pmatrix} 3 & 0 & 4 & 0 \\ 0 & 3 & 0 & 4 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & a \end{pmatrix} = 0.$$

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12. Let $\mathbf{v}_1$ be the vector in R^3 whose entries are the first three digits of your student identification number. Similarly, let $\mathbf{v}_2$ be the vector whose entries are the middle three digits of your identification number, and let $\mathbf{v}_3$ be the vector whose entries are the last three digits of your identification number. Are your vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ linearly independent vectors in R^3? Explain why or why not.