

A. History (2 points)

One of Kirchhoff's laws states that the sum of the currents flowing into any point of an electric circuit is zero. This law was formulated by the German physicist Gustav Robert Kirchhoff. He was born in Königsberg (now Kaliningrad), then part of Prussia and now a Russian enclave sandwiched between Poland and Lithuania.

In addition to working on electric circuits, Kirchhoff laid the foundations of the theory of black body radiation and made fundamental contributions to spectral analysis (partly in collaboration with Bunsen, inventor of the famous burner). His studies initiated a new era in astronomy and were significant for the subsequent development of quantum theory. You can find his masterpiece *Vorlesungen über mathematische Physik* in the Microtext Department on the second floor of the Sterling C. Evans Library on campus.

A disability incurred in an accident forced Kirchhoff to use crutches or a wheelchair. There is a crater on the moon named for Kirchhoff.



G. R. Kirchhoff
1824–1887

- As we discussed during the last week of class, Kirchhoff's name is often misspelled. Write the correct spelling of the name "Kirchhoff" in your blue book.

B. Computation (25 points each)

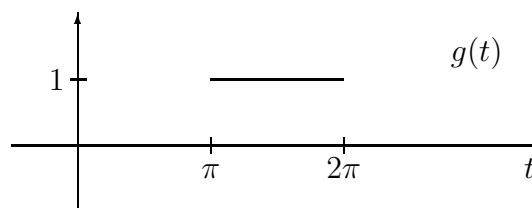
- (This is problem 4 on page 79 of the textbook.) Solve the following initial value problem for a first-order differential equation:

$$\frac{dy}{dx} + \frac{3y}{x} = x^2 - 4x + 3, \quad y(1) = 1.$$

- (This is exercise 34 on page 400 of the textbook.) Solve the following initial value problem for a second-order differential equation:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = g(t), \quad y(0) = 0, \quad \frac{dy}{dt}(0) = 0,$$

$$\text{where (see the diagram) } g(t) = \begin{cases} 0, & 0 \leq t \leq \pi \\ 1, & \pi < t < 2\pi \\ 0, & 2\pi \leq t \end{cases}$$



C. Problem Solving (20 points each)

In the following problems, be sure to define the variables that you employ and to explain any assumptions that you make about the mathematical model.

4. (This is exercise 2 on page 114 of the textbook.) A 400 pound object is released from rest 500 feet above the ground and allowed to fall under the influence of gravity (assume $g = 32 \text{ ft/sec}^2$). Suppose the force in pounds due to air resistance is $-10v$, where v is the velocity of the object in feet per second.
 - (a) Set up a differential equation describing the motion of the object.
 - (b) When will the object hit the ground? (Round the answer to two decimal places.)
5. (This is exercise 33 on page 216 of the textbook.) Consider the temperature of a house on a summer day when the outside temperature is 100°F . Model the house as consisting of two compartments: the living area, which is air conditioned; and the attic, which is not. Suppose that the air conditioner cools the living area at a rate that would decrease its temperature by 12°F per hour if there were no other effects.

Assume the following values for the time constants for heat transfer: between the living area and outside, 4 hours; between the living area and the attic, 4 hours; between the attic and outside, 2 hours. (Recall that the proportionality constant in Newton's law of heating and cooling is the *reciprocal* of the time constant.)

- (a) Set up this scenario in the language of differential equations.
- (b) This (oversimplified!) model predicts that the temperature in the attic will approach a steady-state value. Find this steady-state attic temperature.

D. Theory (8 points)

6. (This is an excerpt from Project D on pages 233–234 of the textbook.) The second-order differential equations we studied were *linear*, for the nonlinear ones rarely admit simple solutions in terms of elementary functions. However, there is a general method for solving equations of the form $y'' = F(y, y')$ in which the independent variable does not appear explicitly.

- (a) Set $w = \frac{dy}{dx}$ and use the chain rule to express $\frac{d^2y}{dx^2}$ in terms of w and $\frac{dw}{dy}$. This substitution converts the equation $y'' = F(y, y')$ into a first-order differential equation in the variables w and y .
- (b) Use this technique to solve the initial value problem

$$y'' + yy' = 0, \quad y(0) = 2, \quad y'(0) = -2.$$

My mother pointed out that Maple's `dsolve` command is unable to find the answer because Maple fails to evaluate the first integration constant before computing the second integral.