

**A. Computation** (20 points each) Solve the following first order differential equations by hand. State what method you are using (for example: linear, separable, exact) and show all steps.

1.  $\frac{dy}{dx} + 3x^2y = 0$

2.  $(x - t) dt + dx = 0$

3. Initial value problem:  $\frac{dy}{dx} - xy = \frac{x}{y}, \quad y(0) = 1$

**B. Problem Solving** (15 points each) In the following problems, the words are as important as the numbers. Define the variables that you employ and explain what assumptions you make to arrive at a mathematical model.

4. Consider the differential equation  $\frac{dx}{dt} = \frac{1}{10} - \frac{x}{1+t}$ . Invent a scenario involving a swimming pool for which this differential equation is an appropriate model. You are not required to solve the equation.

5. When a pie is finished cooking, it is removed from a hot oven at 8:00 PM and set on a kitchen counter to cool. At 8:30 PM, the temperature of the pie is 185°F. At 9:00 PM, the temperature of the pie is 119°F. The temperature of the kitchen is 75°F. What was the temperature of the oven?

**C. Theory** (10 points) You may do this problem either by hand or with the computer. If you choose to use the computer (for example, to draw direction field plots using Maple's `DEplot` command), first hand in your blue book containing your solutions to the other problems.

6. This problem is personalized. Let  $A$  denote your student identification number, expressed as a decimal. (For example, the ID number 987-65-4321 would correspond to  $A = 0.987654321$ .) Let  $B$  denote your group number (which is a whole number between 1 and 12).

Suppose that the population  $P$  of the Puorg marmots in Rebmundi province satisfies the peculiar differential equation  $\frac{dP}{dt} = AP - B \sin(P)$ , where the constants  $A$  and  $B$  are specified in the preceding paragraph. You cannot expect to solve this nonlinear equation with an explicit formula.

Discuss the behavior of the population  $P(t)$  as  $t \rightarrow \infty$ . In particular, how does the long-term behavior of the population depend on the initial value of the population?