A. Computation (25 points each) Solve the following linear second order differential equations, showing all steps.

1. $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 4y = -30\cos(2x)$ with $y(0) = 5$ and $y'(0) = 26$ as initial conditions

2. $\dfrac{d^2y}{dx^2} + y = \sec(x)$ for $-\pi/2 < x < \pi/2$ (find the general solution)

B. Problem Solving (20 points each) In the following problems, be sure to define the variables that you employ and to explain any assumptions that you make about the mathematical model.

3. Suppose that two boxes, one with mass 1 kg and the other with mass 2 kg, lie on a horizontal frictionless surface. The boxes are attached to each other by a spring with spring constant 1 N/m. The heavier box with mass 2 kg is also attached to a wall by a spring with spring constant 2 N/m. The boxes and springs all lie in a straight line.

   (a) Determine a pair of linear second order differential equations describing the motion of this coupled spring-mass system.

   (b) Determine the two natural frequencies of the system.

4. The current $I(t)$ in a series RLC circuit can be modeled by a differential equation $L\dfrac{d^2I}{dt^2} + R\dfrac{dI}{dt} + \dfrac{1}{C}I = \dfrac{dE}{dt}$. Find values for the inductance L, the resistance R, the capacitance C, and the voltage source $E(t)$ so that $I(t) = (5 + 4t)e^{-3t}$ will be a solution of this differential equation.

C. Theory (10 points)

5. Every linear, homogeneous, second order differential equation of the form $y'' + p(x)y' + q(x)y = 0$ can be transformed into a so-called *normal form* $u'' + f(x)u = 0$ having no first-derivative term. The transformation can be achieved by substituting $y = u(x)v(x)$ for a suitable function v.

   Your task is to derive a formula for how to choose v (in terms of p and q).

   (a) Make the substitution $y = u(x)v(x)$ in the differential equation $y'' + p(x)y' + q(x)y = 0$ and determine a first-order differential equation that v must satisfy in order for the u' terms to cancel out.

   (b) Solve for v. Your answer will contain one unevaluated integral.