

- First order differential equations can be used to model many physical processes: population growth, mixing problems, heating and cooling, motion of an object in a straight line. Many of these models are linear differential equations, which can be solved explicitly.
- The simplest model of population growth is the Malthusian law $\frac{dP}{dt} = aP$, where a is a constant. Since this model leads to an exponentially growing solution, the logistic model $\frac{dP}{dt} = aP - bP^2$ is often considered more realistic. The logistic equation is separable and leads to solutions that approach an equilibrium when time increases.
- Newton's law of cooling says that the rate of change of temperature of an object is proportional to the difference between the object's temperature and the temperature of the surroundings. This model gives a linear differential equation whose solution involves a factor e^{-Kt} , and $1/K$ is a characteristic time scale called the time constant.
- Newton's law of motion says that $m\frac{dv}{dt}$ equals the total force, which for a body falling under gravity with resistance would typically be $mg - kv$, where v is the velocity, k is a constant, m is the mass, and g is the gravitational constant. This linear differential equation for the velocity leads to solutions that approach a limiting terminal velocity.
- The above examples are exceptional, as most differential equations cannot be solved by a formula and must be solved numerically.
 - The simplest method for numerically approximating the solution of an initial value problem, Euler's method, is not very accurate: the error is of the same order of magnitude as the step size.
 - Euler's method can be improved by moving along the direction field to predict a new point, computing the slope at the new point, and then correcting the prediction by averaging the slope at the new point with the slope at the old point. This predictor-corrector method has an error of the same order of magnitude as the square of the step size.
 - The Runge-Kutta method is a more elaborate procedure that uses four function evaluations at each step instead of two. It has an error that is of the same order as the fourth power of the step size. The `numeric` option on Maple's `dsolve` command employs a refinement known as the Runge-Kutta-Fehlberg or RKF45 method, which uses a variable step size to help control the error.