Math 311-102

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Announcements

1. No office hour tomorrow morning (Thursday, June 30).
I will, however, be available in my office after class both today and tomorrow.

2. The comprehensive final exam is 1:00–3:00PM, Tuesday, July 5, in this room.

Gauss's theorem

If R is a region in $\mathbb{R}^3$ bounded by a closed surface S, and $\vec{F}$ is a vector field, then $\iint_S \vec{F}\cdot d\vec{S} = \iiint_R (\nabla\cdot\vec{F})\,dV$

($dV = dx\,dy\,dz$ is the volume element, $\nabla\cdot\vec{F}$ is the divergence of $\vec{F}$, and the surface area element $d\vec{S} = \vec{n}\,d\sigma$ is taken with the outward-pointing normal vector).

Example. #12, page 437: Find $\iint_S \vec{F}\cdot d\vec{S}$ when $\vec{F} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ and the surface S has top $z = 1 - x^2 - y^2$ and bottom $x^2 + y^2 \le 1, z = 0$.

Solution. Since $\nabla\cdot\vec{F} = 3$, the integral equals 3 times the volume of the enclosed region. Using cylindrical coordinates gives $3\int_0^{2\pi}\int_0^1\int_0^{1-r^2} r\,dz\,dr\,d\theta = 6\pi\int_0^1 r(1-r^2)\,dr = 3\pi/2$.