Topics in Applied Mathematics I

We did these exercises in groups.

1. (Exercise 3, page 408)

   By Green's theorem, $\oint_\gamma y\,dx + x^2\,dy = \iint_R (2x-1)\,dx\,dy$, where R is the interior of the rectangle. This integral equals $\int_0^1\int_0^1 (2x-1)\,dx\,dy = 0$.

2. (Exercise 7, page 408)

   By Green's theorem, $\oint_\gamma (x-y)\,dx + (x+y)\,dy = \iint_T 2\,dx\,dy$, where T is the interior of the triangle. The answer is therefore twice the area of the triangle, or 1.

3. (Exercise 9, page 408)

   By Green's theorem, the *clockwise* integral $\int_c (x^2-y^2)\,dx + (x^2+y^2)\,dy$ is equal to $-\iint_R (2x+2y)\,dx\,dy$, where R is the interior of the unit circle. By symmetry, this integral is equal to 0.

4. (Exercise 11, page 408)

   By Green's theorem, $\oint_\gamma (-y\,dx + x\,dy) = \iint_D 2\,dx\,dy$, where D is the region inside the simple closed curve γ. The latter integral represents 2 times the area of D. Divide by 2 to get the desired result.