Linear Algebra

Instructions Please answer the five problems on your own paper. These are essay questions: you should write in complete sentences.

1. Recall that P_3 denotes the vector space of polynomials of degree less than 3. Let S denote the two-dimensional subspace of P_3 consisting of polynomials $p(x)$ such that $p(0) = p(1)$. Find a basis for S, and explain how you know that your answer is a basis.

2. Recall that $R^{2\times 2}$ denotes the vector space of all 2×2 matrices with real entries.

   (a) Show that the set of all *symmetric* 2×2 matrices with real entries is a subspace of $R^{2\times 2}$. (Recall that a matrix A is symmetric if $A = A^T$.)

   (b) What is the dimension of this subspace? How do you know?

3. Give an example of a linear transformation $L\colon R^2 \to R^2$ whose kernel equals the span of the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. (There are many correct answers.)

4. Suppose that $\mathbf{u}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. If $L\colon R^2 \to R^2$ is a linear transformation such that $L(\mathbf{u}_1) = \mathbf{u}_1$ and $L(\mathbf{u}_2) = 2\mathbf{u_2}$, find the matrix representation of L with respect to the *standard* basis.

5. Rose and Colin are studying a certain 3×4 matrix A. They use a TI-89 calculator to find the following reduced row echelon forms for the matrix A and for the transpose A^T:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

   Use this information to say as much as you can about the null space, the row space, and the column space of the original matrix A.
   [Can you determine the dimension of each subspace? Can you determine a basis for each subspace?]