

Complex Variables

Instructions Please write your name in the upper right-hand corner of the page. Write complete sentences to explain your solutions.

Reminder When z is a complex variable, the cosine function and the sine function are defined in terms of the exponential function via

$$\cos(z) = \frac{1}{2}(e^{iz} + e^{-iz}) \quad \text{and} \quad \sin(z) = \frac{1}{2i}(e^{iz} - e^{-iz}).$$

1. Determine the real and imaginary parts of $\cos(i^3)$.

2. Show that if $f(z) = \cos(z)$, then $f'(z) = -\sin(z)$ (as you would expect from the corresponding differentiation formula for functions of a real variable).

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3. Evaluate the line integral $\int_{\gamma} |z|^2 dz$, where γ is the line segment from the point 0 to the point $1 + i$.