

Complex Variables

Instructions Please write your solutions on your own paper.

These problems should be treated as essay questions. You should explain your reasoning in complete sentences.

1. State the following:
 - (a) De Moivre's theorem about powers of complex numbers;
 - (b) the Cauchy–Riemann equations.
2. Which of the complex numbers $\left(\frac{1+i}{2}\right)^4$ and $\left(\frac{1}{\sqrt{3}-i}\right)^2$ has bigger imaginary part? Which of these two complex numbers has bigger modulus? Explain how you know.
3. Consider the sequence z_1, z_2, \dots of complex numbers defined recursively as follows:

$$z_1 = 3 + 2i \quad \text{and} \quad z_{n+1} = \frac{i}{z_n} \quad \text{when } n \geq 1.$$

Determine all the limit points of this sequence.

4. Describe geometrically the set of points z in the complex plane satisfying the property that
$$|z - 1| = \text{Im}(z).$$
5. If $f(x + iy) = x^3 - y^3 + 3ix^2y$, is the function f analytic? Explain why or why not.
6. Evaluate the integral $\int_C (\bar{z} - z) dz$, where C is the straight line segment joining the point $(0, 0)$ to the point $(1, 3)$ in the complex plane.

Extra credit

I typed “real part of cube root of -8 ” into WolframAlpha, and I received back the answer 1 instead of the expected value of -2 . Explain what decision Wolfram's programmers must have made that resulted in the computer giving the answer 1.