Examination 1

Instructions Please write your solutions on your own paper. These problems should be treated as essay questions to answer in complete sentences.

1. Suppose z is a complex number such that the imaginary part of z is equal to 7, and the imaginary part of z^2 is equal to 56. Determine the value of the real part of z.

2. The set of values of the complex variable z for which
$$2|z|^2 = |z - i|^2$$
is a circle in the complex plane. Determine the radius of that circle.

3. Show that no complex number z exists for which $\tan(z) = i$.
Hint: Recall that by definition,
$$\tan(z) = \frac{\sin(z)}{\cos(z)}.$$

4. a) State the definition of what the complex derivative $f'(0)$ means (in terms of a limit).
   b) Use the definition to show that if $f(z) = |z|^2$, then the complex derivative $f'(0)$ exists.

5. Suppose $f(z)$ is an analytic function, written as usual in the form $u(x, y) + iv(x, y)$ in terms of real functions u and v.
   a) State the Cauchy–Riemann equations.
   b) Suppose now that $u(x, y) = 2v(x, y)$ for all values of the real variables x and y, and $u(0, 0) = 2$. Determine $f(z)$.

6. Determine a Möbius transformation (fractional linear transformation) that maps the three points -1, 0, and 1 to the image points 0, -1, and 1 (in that order).

Extra Credit

Zed and Zee have a debate about the value of $\lim_{z\to 0} z^z$, where z is a complex variable. Zed says, "0 raised to any power equals 0, so the answer must be 0." Zee says, "Any number raised to the power 0 equals 1, so the answer must be 1."
Do you agree with either Zed or Zee? What do you think about the value of this limit? Explain your reasoning.