Welcome

Math 407
Complex Variables
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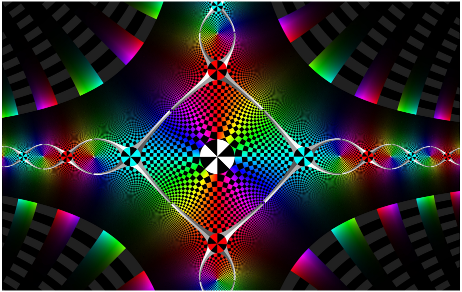
What do we remember about calculus?

- chain rule for derivatives
- trigonometric identities
- intermediate-value theorem
- integration by parts
- Riemann sums
- multiple integrals
- differential equations
- partial derivatives
- Gauss's theorem, Green's theorem, Stokes's theorem

Examples you will learn how to explain

- $\int_0^\infty \frac{1}{x(1+x^2)} \log \left| \frac{x+\sqrt{3}}{x-\sqrt{3}} \right| \, dx = \frac{\pi^2}{6}$
- What is going on in this picture?



Example of an unsolved problem about complex variables

For which values of the complex variable z is $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^z} = 0$?

Riemann hypothesis: When $0 < \mathrm{Re}(z) < 1$, every solution has real part equal to $1/2$.

Bernhard Riemann
(1826–1866)

What are the complex numbers?

$ax^2 + bx + c = 0$ has solutions $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (quadratic formula)

We create an "imaginary" number i with the property that $i^2 = -1$.

In general, complex numbers have the form $a + bi$, where a and b are real numbers.

Examples: $2 + 3i$, $4 - 7i$, 3, $-4i$

Example computations

1. Solve $\dfrac{1}{3-4i} = a + bi$.
   Method 1: multiply and divide by $3+4i$:

$$\frac{1}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{3+4i}{3^2-(4i)^2} = \frac{3+4i}{25} = \frac{3}{25} + \frac{4}{25}i$$

   Method 2: Clear the denominator to get the equivalent equation $1 = (3-4i)(a+bi) = 3a - 4ia + 3bi - (4i)(bi) = 3a + 4b + i(-4a+3b)$. This leads to simultaneous equations $1 = 3a + 4b$ and $0 = -4a + 3b$.

2. Solve $\sqrt{3-4i} = a + bi$.
   Strategy: start by squaring both sides: $3 - 4i = (a+bi)^2$.
   Answer: $-2 + i$ and $2 - i$.
   Check: $(-2+i)^2 = 3 - 4i$.

Assignment

- Read section I.1 in the textbook.
- Find an exercise at the end of section I.1 that you don't know how to solve.