Structure of $\mathbb{C}$, the complex numbers

- Algebraic structure: $\mathbb{C}$ is a *field*.
  We can add, subtract, multiply, divide (except by 0), and the commutative, associative, and distributive laws hold.
- In contrast to $\mathbb{R}$ (the real numbers), the field $\mathbb{C}$ is *algebraically closed*.
  $x^2 + 1 = 0$ has no solution in $\mathbb{R}$ but does have a solution in $\mathbb{C}$.
  In fact, every polynomial has a root in the complex numbers.
- In contrast to $\mathbb{C}$, the field $\mathbb{R}$ is *ordered*.
  When we write inequalities in this course, they have to involve absolute values of complex numbers.
- Metric structure: there is a distance function on $\mathbb{C}$, so we can talk about *limits*.

Polar representation

A point (x, y) in $\mathbb{R}^2$ can be written $(r\cos(\theta), r\sin(\theta))$ in polar coordinates.
In complex notation, $z = x + iy = r(\cos(\theta) + i\sin(\theta)) = re^{i\theta}$ by Euler's formula.

Example

Compute $(1+i)^{407}$.
Solution: $1 + i = \sqrt{2}\, e^{i\pi/4}$, so
$(1+i)^{407} = 2^{407/2} e^{407 i\pi/4} = 2^{407/2} e^{7i\pi/4}$ since $e^{400 i\pi/4} = 1$.
But $e^{7i\pi/4} = \frac{1-i}{\sqrt{2}}$, so the final answer is $2^{203}(1-i)$.

Assignment to hand in next time

- Section I.5: 1(b),(d)
- Section I.2: 1(c),(h)
- Section I.1: 1(b)