Reminder

Exam 2 takes place on Thursday, October 26.

The material covered is Sections 2–7 of Chapter II.

Please bring your own paper to work on.

Recap: The group of fractional linear transformations or linear fractional transformations or Möbius transformations

Compositions of translations, rotations, dilations, and inversion ($z \mapsto 1/z$) yield transformations of the form

$$z \mapsto \frac{az+b}{cz+d},$$

where a, b, c, and d are complex numbers, and $ad - bc \neq 0$.

The "point at infinity"

We invent a point called ∞ and declare that $1/0 = \infty$ and $1/\infty = 0$.

Then we can think of the fraction $\dfrac{az+b}{cz+d}$ as being defined on $\mathbb{C} \cup \{\infty\}$: namely, the complex number $-d/c$ maps to the point ∞, and ∞ maps to the complex number a/c.

Group quiz

Suppose $w = \dfrac{z-1}{z+1}$.

1. Find the image in the w plane of the following points in the z plane: 0, 1, ∞, i.

2. Find the image in the w plane of the set $\{ z : \mathrm{Re}(z) = 0 \}$ (the vertical axis in the z plane).
   The answer is some circle: which circle?

3. Find the image in the w plane of $\{ z : |z| = 1 \}$ (the unit circle in the z plane).