Recap of Cauchy's integral formula

If f is analytic on and inside a closed curve C, and b is a point inside C, then

$$f(b) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z - b}\, dz.$$

More generally,

$$f^{(n)}(b) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z - b)^{n+1}}\, dz,$$

where $f^{(n)}$ means the nth derivative.

Memories of infinite series?

- Taylor series
- ratio test
- if the terms do not have limit zero, then the series diverges
- geometric series

Main convergence tests

Each one of the following conditions is *sufficient* for the series $\sum_{n=0}^{\infty} c_n$ to converge (absolutely).

- There is a convergent series $\sum_n a_n$ of positive numbers such that $|c_n| \leq a_n$ for all (sufficiently large) n.
  [comparison test]
- $\lim_{n\to\infty} \left| \frac{c_{n+1}}{c_n} \right| < 1$. [ratio test]
- $\lim_{n\to\infty} |c_n|^{1/n} < 1$. [root test]

Assignment

- Parts b, c, and f of Exercise 1 in Section V.3.