Quiz, September 26

1. Find a complex number z with the property that $e^{iz} = 2i$.

   Solution. One method is to take logarithms to see that $iz = \log(2i) = \ln 2 + i(\frac{\pi}{2} + 2n\pi)$, where n is an arbitrary integer. Multiplying through by $-i$ shows that $z = -i\ln 2 + \frac{\pi}{2} + 2n\pi$. Since the problem does not ask for the most general solution, you could specialize to the principal value, which is $\frac{\pi}{2} - i\ln 2$.

2. Show that $\sin(iz) = i\sinh(z)$ for all values of the complex variable z.

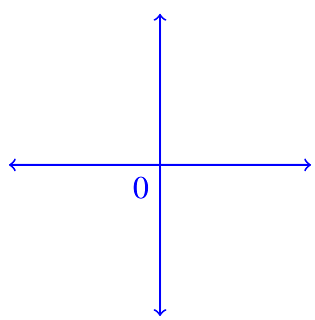
   Solution. The representation of the sine function in terms of the exponential function shows that
   $$\sin(iz) = \frac{e^{i\cdot iz} - e^{-i\cdot iz}}{2i} = \frac{e^{-z} - e^{z}}{2i} = \left(\frac{-1}{i}\right)\left(\frac{e^{z} - e^{-z}}{2}\right).$$
   But $-1/i = i$, so the representation of the hyperbolic sine function in terms of the exponential function shows that the right-hand side is equal to $i\sinh(z)$, as required.

   Remark. This formula is one of many in Section I.8 of the textbook.

3. Draw a picture of the set of values of the complex variable z for which $|z| = |\operatorname{Re} z| + |\operatorname{Im} z|$.

   Solution. Write z as $x + yi$. Then the equation says that $\sqrt{x^2 + y^2} = |x| + |y|$. Both sides are positive real numbers (or possibly 0), so squaring both sides gives an equivalent equation:
   $$x^2 + y^2 = |x|^2 + 2|x|\,|y| + |y|^2.$$
   But x and y are *real* numbers, so $|x|^2 = x^2$, and $|y|^2 = y^2$. Canceling the common terms shows that $0 = 2|x|\,|y|$.

   A product is equal to 0 precisely when one of the factors is equal to 0, so either $x = 0$ or $y = 0$ (or both). The solution set therefore consists of all the points on either coordinate axis. You could draw the picture as follows:

   Remark. This problem is part of Exercise I.1.4 in the textbook.