Advanced Calculus I

Second Examination

1. (a) i. Define what it means for a function to be strictly increasing.
   ii. Give an example of a function that is strictly increasing.
   iii. Give an example of a function that is not strictly increasing.

   (b) i. Define what it means for an interval to be compact.
   ii. Give an example of an interval that is compact.
   iii. Give an example of an interval that is not compact.

2. (a) State the alternating series test (called "Cauchy's test for alternating series" in the book).

   (b) State the Intermediate Value Theorem.

3. (a) State the definition of "the function f is continuous at the point x_0" in the form "for every $\epsilon > 0 \ldots$".

   (b) Use the definition in part (a) to prove that the function $f(x) = 1/x$ is continuous at the point 1.

4. For each of the following statements, say whether the statement is true or false. To support an answer of "true", you must give an explanation or cite a theorem or supply a proof; to support an answer of "false", you must exhibit a counterexample.

   (a) If a function f is locally bounded on an interval, then f is bounded on the interval.

   (b) If a function g has a jump discontinuity at 0, and a function h is continuous at 0, then the product function gh has a jump discontinuity at 0.

5. Determine the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1+2^n}{1+n^2}\right) x^n$.

   Indicate what theorem(s) you are using.