Math 409-502

Harold P. Boas
boas@tamu.edu

Announcement

Math Club Meeting
Monday, October 18 (today)
6:00 PM, Blocker 156

Undergraduate speakers:
Dakota Blair, "Oscillating Patterns in Langtons Ant"
Ryan Westbrook, "New Results in Wavelet Set Theory"

FREE FOOD

Limits of functions

Definition

$\lim_{x\to a} f(x) = L$ means that for every $\epsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \epsilon$ when $0 < |x - a| < \delta$.
(Note that $f(a)$ need not be defined.)

Example: Prove that $\lim_{x\to 1} \frac{x+3}{x+1} = 2$.

Suppose $\epsilon > 0$ is given. Set $\delta = \min(\epsilon, 1)$. If $|x - 1| < \delta$, then in particular $x > 0$, so $\frac{1}{x+1} < 1$.
Hence $|x - 1| < \delta$ implies

$$\left|\frac{x+3}{x+1} - 2\right| = \frac{|-x+1|}{x+1} \le |x-1| < \delta \le \epsilon.$$

Thus $\frac{x+3}{x+1} \underset{\epsilon}{\approx} 2$ when $x \underset{\delta}{\approx} 1$, as required.

Connection with sequences

$\lim_{x\to a} f(x) = L$ if and only if for every sequence $\{x_n\}$ such that $x_n \to a$ [but $x_n \neq a$] we have $f(x_n) \to L$.

Consequently, all our theorems for limits of sequences carry over to limits of functions.

Example: $\lim_{x\to 0} x\sin(1/x) = 0$

Proof: since

$$-|x| \leq x\sin(1/x) \leq |x| \qquad \text{(for } x \neq 0\text{)}$$

the result follows from the squeeze theorem.

Continuity

A function f is *continuous* at a point a if $\lim_{x\to a} f(x) = f(a)$.

The formal definition: For every $\epsilon > 0$, there exists $\delta > 0$ such that $|f(x) - f(a)| < \epsilon$ when $|x - a| < \delta$.

The same proof we did to show that $\lim_{x\to 1} \frac{x+3}{x+1} = 2$ proves that the function $f(x) = \frac{x+3}{x+1}$ is continuous at $x = 1$.

Homework

1. Read sections 11.1 and 11.2, pages 151–158.
2. Do Exercises 11.1/5 and 11.2/1 on page 167.