

Math 409-502

Harold P. Boas
boas@tamu.edu

Reminder

Second examination is Monday, November 1.

The exam covers material through section 13.4.

Intermediate-value property

A function f defined on an interval $[a, b]$ has the intermediate-value property if every value between $f(a)$ and $f(b)$ is in the range of f .

Every continuous function has the intermediate-value property on each interval in its domain, but not every function with the intermediate-value property is continuous.

Example

$$f(x) = \begin{cases} \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

This f is continuous on every interval not containing 0, and on every interval containing 0, the range of f is $[-1, 1]$.

Monotonic functions and continuity

Theorem: If f is strictly increasing on an interval $[a, b]$, then the following properties are equivalent.

1. f is continuous.
2. f has the intermediate-value property.
3. f has a continuous inverse function on $[f(a), f(b)]$.
4. f has an inverse function that satisfies the intermediate-value property.

Property 1 \Rightarrow property 2 by the Intermediate Value Theorem.

Property 2 \Rightarrow property 3 because the only discontinuities of monotonic functions are jump discontinuities.

Property 3 \Rightarrow property 4 by the Intermediate Value Theorem.

Property 4 \Rightarrow property 1 for the same reason that property 2 \Rightarrow property 3.

Homework

- Read sections 12.3 and 12.4 (pages 178–180) and sections 13.1 and 13.2 (pages 185–187).
- Do Exercise 12.4/2 on page 181 and Exercise 13.1/1a,b on page 192.