Math 409-502

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Announcement

Meeting for undergraduate mathematics majors

Tuesday, September 28

7:15pm in Blocker 102

FREE PIZZA and soft drinks

Reminder

First Examination

Friday, October 1

Exam covers Chapters 1–6 and Sections 7.1–7.2

Convergence tests for infinite series

Comparison test (positive terms)

Suppose that $0 \le a_n \le b_n$ for all (large) n.
If $\sum_n b_n$ converges, then $\sum_n a_n$ converges too.
Conversely, if $\sum_n a_n$ diverges, then $\sum_n b_n$ diverges too.

Examples

Use comparison to test for convergence

$$\text{(i)} \quad \sum_{n=3}^{\infty} \frac{\ln(n)}{n} \qquad\qquad \text{(ii)} \quad \sum_{n=3}^{\infty} \frac{\ln(n)}{2^n}$$

Convergence tests (continued)

n-th term screening test

If $a_n \not\to 0$, then $\sum_n a_n$ diverges
(but the converse is not true).

Examples

- $\sum_n \cos(n)$ diverges because $\cos(n) \not\to 0$.
- Although $1/n \to 0$, nonetheless $\sum_n 1/n$ diverges (harmonic series).

Convergence tests (continued)

Root test (positive terms)

If $0 \le a_n$, and if $\sqrt[n]{a_n} \to L$, then

- $\sum_n a_n$ converges when $L < 1$
- $\sum_n a_n$ diverges when $L > 1$
- the root test gives no information when $L = 1$

Examples

Test for convergence

$$\text{(i)} \quad \sum_n \frac{n^2}{2^n} \qquad\qquad \text{(ii)} \quad \sum_n \frac{(1+\frac{1}{n})^n}{n^2}$$

Root test (continued)

Proof of the root test

Suppose $\sqrt[n]{a_n} \to L < 1$. Choose a number r such that $L < r < 1$ (for instance, $r = \frac{1}{2}(L+1)$).

Then $\sqrt[n]{a_n} < r$ when n is sufficiently large,
so $a_n < r^n$ when n is sufficiently large.

Comparing the tail of the series $\sum_n a_n$ with the tail of the convergent geometric series $\sum_n r^n$ shows that $\sum_n a_n$ converges.

Homework

- Read sections 7.3–7.4, pages 100–104.
- In preparation for the examination, make a list of the main definitions and theorems in the course so far.