Examination 2

Please write your solutions on your own paper. These problems should be treated as essay questions to answer in complete sentences.

Students in Section 501 should answer questions 1–6 in Parts A and B.

Students in Section 200 should answer questions 1–3 in Part A and questions 7–9 in Part C.

Part A, for both Section 501 and Section 200

1. Show that the function e^{-x} has exactly one fixed point. In other words, there is exactly one real number x with the property that $e^{-x} = x$.

2. Give an example of a continuous function $f : \mathbb{R} \to \mathbb{R}$ that is a one-to-one correspondence (in other words, has an inverse function) but is not differentiable.

3. Suppose $f(x) = \tan(\sin(\cos(x)))$. Is there a value of x for which $f'(x) = 0$? Explain why or why not.

Part B, for Section 501 only

4. Give an example of a strictly increasing, bounded, continuous function $f : \mathbb{R} \to \mathbb{R}$.

5. Does the limit
$$\lim_{x\to 0} \frac{\sin(x)\cos(x^2)}{|x|}$$
exist? Explain why or why not.

Examination 2

6. The following table has two missing entries: $f'(1)$ and $f'(2)$.

x	$f(x)$	$f'(x)$
1	2	
2	3	
3	4	5

Determine the two missing values if

$$(f \circ f)'(1) = 6, \quad \text{and}$$
$$(f^{-1})'(3) = 7.$$

Remember that the notation f^{-1} means the inverse function, not the reciprocal function.

Part C, for Section 200 only

7. Give an example of a differentiable, convex function $f : \mathbb{R} \to \mathbb{R}$ for which the second derivative $f''(0)$ does not exist.

8. Let $\lceil x \rceil$ denote the ceiling function: namely, the smallest integer greater than or equal to x. Discuss

$$\limsup_{x\to\infty} \frac{\lceil x \rceil \, \cos(x)}{\sqrt{1+x^2}}.$$

9. Show by a counterexample that the analogue of the chain rule fails for right-hand derivatives: namely, there exist continuous functions f and g that do have right-hand derivatives, yet

$$(f \circ g)'_+(0) \neq f'_+(g(0))g'_+(0).$$