Example of making a mistake

$$
\begin{aligned}
0 &= (1-1)+(1-1)+(1-1)+\cdots \\
&\qquad \text{(regroup by associativity of addition)} \\
&= 1+(-1+1)+(-1+1)+\cdots \\
&= 1.
\end{aligned}
$$

What went wrong?
Something is hiding in the $\cdots$. We need to make the limit concept precise.

Introductions

- Who are you?
- What do you do for fun?
- When will you graduate?
- Where are you from?
- Why are you studying mathematics?

What are the real numbers?

Examples: rational numbers like $5/3$ and $-34/7$. Also irrational numbers like $\sqrt{2}$ and π.

More generally, decimal expansions like $382.765\ldots$.

Abstractly, the real numbers are a complete, ordered field.

Fields

A field is a number system with two operations, called $+$ and $\times$, that are commutative and associative. Multiplication distributes over addition. There is an additive identity element, called 0. There is a multiplicative identity element, called 1. Every element has an additive inverse. Every nonzero element has a multiplicative inverse. Also, $1 \neq 0$.

Some examples of fields

- the real numbers $\mathbb{R}$
- the rational numbers $\mathbb{Q}$
- the field with two elements $\{0, 1\}$ with $1 + 1 = 0$

Some non-examples of fields

- the set of integers $\mathbb{Z}$ (missing multiplicative inverses)
- the natural numbers 0, 1, 2, . . . , denoted $\mathbb{N}$ (missing both additive inverses and multiplicative inverses)

Ordered fields

An ordered field has a distinguished subset P, called the positive elements, closed under addition and multiplication. Moreover, every nonzero element of the field either is in P or its additive inverses is in P.
Then saying that $a < b$ means that $b - a \in P$.

Some examples of ordered fields

- the real numbers
- the rational numbers

Some non-examples of ordered fields

- the field with two elements $\{0, 1\}$.
  Since $1 + 1 = 0$, the set of what you might think are positive elements fails to be closed under addition.

Assignment for next time

Exercises 1 and 2 on page 6.