Announcements

- The last class meeting is Thursday, April 27.
- The comprehensive final examination takes place on Thursday, May 4, from 12:30 to 2:30 in the afternoon, here in this room.

  As usual, please bring your own paper to the exam.
- Next week, I will hold my usual office hour on Monday and Wednesday afternoons from 2:00 to 3:00.

Cauchy's theorem on the integral

If f is a continuous function on a compact interval $[a, b]$, then f is integrable in the following sense.

There is a real number I (the value of the integral) such that for every positive tolerance ε, there exists a positive δ with the following property: for every partition of $[a, b]$ into subintervals $[x_0, x_1]$, $[x_1, x_2]$, $\ldots$, $[x_{n-1}, x_n]$ (where $x_0 = a$ and $x_n = b$), each of length less than δ, and for every choice of t_k in the kth subinterval $[x_{k-1}, x_k]$, the difference $I - \sum_{k=1}^{n} f(t_k)(x_k - x_{k-1})$ has absolute value less than ε.

Exercise

Apply Cauchy's theorem on the integral to show that

$$\int_1^b x^{-1/2}\,dx = 2\sqrt{b} - 2$$

by taking t_k to be $\left(\frac{\sqrt{x_k}+\sqrt{x_{k-1}}}{2}\right)^2$.

Properties of the integral

- Linearity: the integral of a sum of two functions is equal to the sum of the integrals, and the integral of a number times a function equals the number times the integral of the function.
- Preservation of order: if $f(x) \leq g(x)$ for every x in interval $[a, b]$, then $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$.
- Absolute value: $\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$.
- Interval decomposition: $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$.