

# Questions

- ▶ boundedness
- ▶ completeness
- ▶ ordered fields

## Variations of the notion of *order*

A *weak* order, usually written  $\leq$ , is a relation that is

- ▶ reflexive:  $x \leq x$  for every  $x$
- ▶ antisymmetric: if  $x \leq y$  and  $y \leq x$ , then  $x = y$
- ▶ transitive: if  $x \leq y$  and  $y \leq z$ , then  $x \leq z$

A *strict* order, usually written  $<$ , is a relation that is

- ▶ irreflexive: there is no  $x$  for which  $x < x$
- ▶ antisymmetric: if  $x < y$ , then it is not the case that  $y < x$
- ▶ transitive: if  $x < y$  and  $y < z$ , then  $x < z$

In Definition 1.1.1 in the textbook, the order is assumed to be a strict order that additionally satisfies the trichotomy property: for every  $x$  and  $y$ , either  $x < y$  or  $y < x$  or  $x = y$  (exclusive “or”).

# Fields

Examples.

$\mathbb{R}$ , the real numbers.

$\mathbb{Q}$ , the rational numbers.

$\mathbb{C}$ , the complex numbers.

Integers modulo a prime number.

Non-examples.

$\mathbb{Z}$ , the integers. (Multiplicative inverses are missing.)

$\mathbb{N}$ , the natural numbers. (Missing both additive inverses and multiplicative inverses.)

## Assignment due next class

- ▶ Write solutions to Exercises 1.1.3 and 1.1.5.
- ▶ Read subsection 1.2.1 in the textbook.