Supremum and infimum, maximum and minimum

Examples

- $E = \left\{ \frac{(-1)^n}{n} : n \in \mathbb{N} \right\}$
  $\sup(E) = 1/2$, $\inf(E) = -1$
  If $\sup(E)$ is an element of E (as it is in this example), you may write $\max(E)$; similarly for inf and min.
- $E = \{\arctan(x) : x \in \mathbb{R}\}$
  Based on the graph, $\sup(E) = \pi/2$, and $\inf(E) = -\pi/2$.
  Here we may not write max and min, for the bounds are not elements of the set E.
- Exercise 1.1.10

Characterization of $\mathbb{R}$

$\mathbb{R}$ is the unique complete, ordered field
(the unique ordered field having the least-upper-bound property).

Cardinality

Two sets have the same *cardinality* when their elements can be put into one-to-one correspondence with each other.

Example

The set of positive rational numbers and the set $\mathbb{N}$ of natural numbers have the same cardinality. Here is a bijection.

$$\text{example:} \qquad \frac{3^2 \times 7^5 \times 19^{50}}{2^4 \times 11^9} \mapsto 2^8 \times 11^{18} \times 3^3 \times 7^9 \times 19^{99};$$

$$\text{general formula:} \qquad \frac{\prod_{j=1}^{*} p_j^{m_j}}{\prod_{k=1}^{**} q_k^{n_k}} \mapsto \prod_{k=1}^{**} q_k^{2n_k} \prod_{j=1}^{*} p_j^{2m_j - 1}$$

Assignment due next class

- Write a solution to Exercise 1.1.6.
- Read the rest of section 1.2 in the textbook.