# lim sup, lim inf, and lim [Theorem 2.3.5]

Theorem

$\lim_{n\to\infty} x_n$ *exists if and only if* $\limsup_{n\to\infty} x_n = \liminf_{n\to\infty} x_n$.

The idea: limit corresponds to the inequality $L - \varepsilon < x_n < L + \varepsilon$.

lim sup addresses the right-hand inequality,
and lim inf addresses the left-hand inequality.

# “I’m lit” exercise

Definition of limit: $\forall \varepsilon > 0 \ \exists M$ such that $\forall n \geq M \ \ |x_n - L| < \varepsilon$.
What do the following inebriated versions say about a sequence?

1. $\forall \varepsilon > 0 \ \ \exists M \ \ \exists n \geq M$ such that $|x_n - L| < \varepsilon$
2. $\forall \varepsilon > 0 \ \ \forall M \ \ \exists n \geq M$ such that $|x_n - L| < \varepsilon$
3. $\forall \varepsilon > 0 \ \ \forall M \ \ \forall n \geq M \ \ |x_n - L| < \varepsilon$
4. $\exists \varepsilon > 0$ such that $\forall M \ \ \exists n \geq M$ such that $|x_n - L| < \varepsilon$
5. $\exists \varepsilon > 0$ such that $\forall M \ \ \forall n \geq M \ \ |x_n - L| < \varepsilon$
6. $\exists \varepsilon > 0 \ \ \exists M$ such that $\forall n \geq M \ \ |x_n - L| < \varepsilon$
7. $\exists \varepsilon > 0 \ \ \exists M \ \ \exists n \geq M$ such that $|x_n - L| < \varepsilon$
8. $\exists M$ such that $\forall \varepsilon > 0 \ \ \forall n \geq M \ \ |x_n - L| < \varepsilon$

# Answers

1. Either some term of the sequence equals $L$, or there exists a subsequence that converges to $L$.
2. There exists a subsequence that converges to $L$.
3. The sequence is the constant sequence $L, L, \ldots$.
4. There exists a bounded subsequence.
5. The sequence is bounded.
6. The sequence is bounded.
7. This property holds for every sequence.
8. The sequence is eventually constant.

# Assignment due next class

Finish reading sections 2.3–2.4 in the textbook.