Reminder

The second exam takes place on Wednesday, March 28.

Material covered: sections 2.3, 2.4, 2.5, 2.6.1, 2.6.2, 3.1.

Introduction to the next topic: Continuous functions

A function f is called *continuous* when it preserves convergent sequences: namely,

$$\lim_{n \to \infty} f(x_n) = f\left(\lim_{n \to \infty} x_n\right).$$

Reminders on function terminology

- domain of a function: the set of all the inputs
- codomain of a function: the target space
- range of a function: the set of all the outputs

Example

$f\colon [-\pi, \pi] \to \mathbb{R}$ defined by $f(x) = \sin(x)$.
The domain is the closed interval $[-\pi, \pi]$, the codomain is $\mathbb{R}$, the range is the closed interval $[-1, 1]$.

In this course, usually the domain is a subset of $\mathbb{R}$, the range is a subset of $\mathbb{R}$, the codomain is $\mathbb{R}$.

Terminology for points and sets

- A point p of a set S is an *isolated* point of S if there is some neighborhood $(p-\varepsilon, p+\varepsilon)$ that contains no other point of S.
  Example: If $S = \mathbb{N}$, then every point of S is isolated.
  Example: If $S = \mathbb{Q}$, then no point of S is isolated.
- A point of S that is not isolated is a *cluster point*.
  More generally, a point p that might or might not belong to S is called a cluster point of S if p is not an isolated point of $S \cup \{p\}$.
  Example: If $S = \mathbb{Q}$, then every real number is a cluster point of S.

Synonyms for cluster point: accumulation point, limit point.

Equivalent formulations of the notion of cluster point

A point p is a cluster point of set S if

- every neighborhood of p contains some point of S different from p, or
- for every positive ε, there exists some x in S such that $x \neq p$ and $|x - p| < \varepsilon$, or
- there exists a sequence $\{x_n\}_{n=1}^{\infty}$ such that $\lim_{n\to\infty} x_n = p$, and $x_n \in S$ for every n, and $x_n \neq p$ for every n.

Example

If S is the open interval $(0, 1)$, then the cluster points of S are all points of the closed interval $[0, 1]$.

Assignment due next class

- Read sections 3.1.1 and 3.1.2 in the textbook.
- If I were to put Exercise 2.3.9 on the exam, would you be happy?