Applied Algebra

Instructions Please write your name in the upper right-hand corner of the page. Use complete sentences, along with any necessary supporting calculations, to answer the following questions.

1. Find a positive, two-digit integer n such that both $\gcd(n, 45) = 9$ and $\gcd(n, 16) = 4$.

   Solution. Since $45 = 5 \times 9$, the condition that $\gcd(n, 45) = 9$ implies that n is a multiple of 9 but not a multiple of 5. Similarly, the condition that $\gcd(n, 16) = 4$ implies that n is a multiple of 4 but not a multiple of 8. Consequently, the required integer n is a multiple of 4×9 but not a multiple of 8×9. Since n is a two-digit integer, the value of n must be 36.

2. Find integers s and t such that $6s + 160t = 2$.

   Solution. The solution can be obtained from the Euclidean algorithm as follows.

   $$\left.\begin{aligned} 160 &= 6 \times 26 + 4 \\ 6 &= 4 \times 1 + 2 \\ 4 &= 2 \times 2 + 0 \end{aligned}\right\} \quad \text{and back substitute:} \quad \left\{\begin{aligned} 2 &= 6 - 4 \\ &= 6 - (160 - 6 \times 26) \\ &= 6 \times 27 + 160 \times (-1). \end{aligned}\right.$$

   Therefore $s = 27$ and $t = -1$.

   Alternatively, you could arrive at the same result via the matrix method, as follows.

   $$\begin{pmatrix} 1 & 0 & 160 \\ 0 & 1 & 6 \end{pmatrix} \xrightarrow{R1 \to R1 - 26R2} \begin{pmatrix} 1 & -26 & 4 \\ 0 & 1 & 6 \end{pmatrix} \xrightarrow{R2 \to R2 - R1} \begin{pmatrix} 1 & -26 & 4 \\ -1 & 27 & 2 \end{pmatrix}.$$

   From the bottom row of the final matrix, you can read off that (as before) $160 \times (-1) + 6 \times 27 = 2$.

   Remark The answer is not unique. You can add $160k$ to s (where k is an arbitrary integer) and simultaneously subtract $6k$ from t to get another valid answer.