Two English words derived from the Latin word *basis*

	meaning	plural	phonetic plural
base	a supporting part	bases	bay-sez
basis	a fundamental part	bases	bay-seez

Basis for a topology

If (X, τ) is a topological space, then a collection $\mathcal{B}$ of open sets is a *basis* for τ when *every* open set is a union of members of $\mathcal{B}$.

Example

One basis for the Euclidean topology on $\mathbb{R}$ is the collection of intervals $\{ (a, b) : a \in \mathbb{R} \text{ and } b \in \mathbb{R} \text{ and } a < b \}$.

Another basis for the Euclidean topology on $\mathbb{R}$ is the collection of intervals $\{ (a, b) : a \in \mathbb{Q} \text{ and } b \in \mathbb{Q} \text{ and } a < b \}$.

Product topology (see Exercises 2.2: #6)

Examples

- If τ is the Euclidean topology on $\mathbb{R}$, then the product topology on $\mathbb{R} \times \mathbb{R}$ is the Euclidean topology on $\mathbb{R}^2$ (open rectangles form a basis).
- If X is equipped with the discrete topology, then what is the product topology on $X \times X$?
  Answer: the discrete topology.
- If $\mathbb{N}$ is equipped with the initial segment topology, then what is the product topology on $\mathbb{N} \times \mathbb{N}$?
  Answer: staircases.

Assignment due next class

1. If $\mathbb{N}$ is equipped with the finite-closed topology, is the corresponding product topology on $\mathbb{N} \times \mathbb{N}$ equal to the finite-closed topology on $\mathbb{N} \times \mathbb{N}$? Why or why not?

2. Prove that if (X, τ_1) is a T_1 space, and (Y, τ_2) is a T_1 space, and τ_3 is the product topology on the Cartesian product $X \times Y$, then $(X \times Y, \tau_3)$ is a T_1 space.

3. Read section 2.3 in the textbook.